Weiss oscillations in the magnetoconductivity of modulated graphene bilayer

M. Tahir*

Department of Physics, University of Sargodha, Sargodha 40100, Pakistan

K. Sabeeh

Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan

Abstract

We present a theoretical study of Weiss oscillations in magnetoconductivity of bilayer graphene. Bilayer graphene in the presence of a perpendicular magnetic field and a unidirectional weak electric modulation is considered.We determine the σ_{yy} component of the magnetoconductivity tensor for this system which is shown to exhibit Weiss oscillations. We show that Weiss oscillations in the magnetoconductivity of bilayer graphene are enhanced and more robust with temperature compared to those in conventional two-dimensional electron gas systems whereas they are less robust with temperature compared to monolayer graphene. In addition, we also find phase differences of π and 2π in the magnetoconductivity oscillations compared to monolayer graphene and conventional 2DEG system which arises due to the chiral nature of quasiparticles in graphene.

I. INTRODUCTION

The successful preparation of monolayer graphene has allowed the possibility of studying the properties of electrons in graphene [1]. The nature of quasiparticles called Dirac electrons in these two-dimensional systems is very different from those of the conventional two-dimensional electron gas (2DEG) realized in semiconductor heterostructures. Graphene has a honeycomb lattice of carbon atoms. The quasiparticles in graphene have a band structure in which electron and hole bands touch at two points in the Brillouin zone. At these Dirac points the quasiparticles obey the massless Dirac equation. In other words, they behave as massless Dirac particles leading to a linear dispersion relation $\epsilon_k = vk$ (with the characteristic velocity $v \simeq 10^6 m/s$). This difference in the nature of the quasiparticles in graphene from conventional 2DEG has given rise to a host of new and unusual phenomena such as anamolous quantum Hall effects and a π Berry phase[1][2]. Besides the fundamental interest in understanding the electronic properties of graphene there is also serious suggestions that it can serve as the building block for nanoelectronic devices [3].

In addition to the graphene monolayer, there has been a lot of interest in investigating the properties of bilayer graphene. The quasiparticles in bilayer graphene exhibit a parabolic dispersion relation which implies that they are massive particles. These quasiparticles are also chiral and are described by spinor wavefunctions[2, 4, 5, 6, 7]. Recent theoretical work on graphene multilayers has also shown the existance of Dirac electrons with a linear energy spectrum in monolayer graphene and a parabolic spectrum for Dirac electrons in bilayer[4]. Bilayer graphene consists of two monolayers stacked as in natural graphite. This, Bernal stacking, yields a unit cell of four atoms with the result that there are four electronic bands. In k space, the bilayer has a hexagonal Brillouin zone. Its physical properties are mainly determined by the eigenvalues and eigenfunctions at two inequivalent corners of the Brillouin zone, K and K', where the π^* conduction and π valence bands meet at the Fermi surface. Due to the strong interlayer coupling both the conduction and valence bands are split by an energy $\sim 0.4eV$ near the K and K' valleys. Only two of these bands, upper valence and lower conduction band, are relevant at low energy and they can be described by the effective Hamiltonian given below[2, 5, 6]

It was found years ago that if conventional 2DEG is subjected to artificially created periodic potentials it leads to the appearence of Weiss oscillations in the magnetoresistance.

This type of electrical modulation of the 2D system can be carried out by depositing an array of parallel metallic strips on the surface or through two interfering laser beams [8, 9, 10]. Weiss oscillations were found to be the result of commensurability of the electron cyclotron diameter at the Fermi energy and the period of the electric modulation. These oscillations were found to be periodic in the inverse magnetic field [8, 9, 10]. Recently, an investigation of electric field modulation effects on transport properties in monolayer graphene has led to the prediction of enhanced Weiss oscillations in the magnetoconductivity[11]. In addition, the magnetoplasmons spectrum, density of states, bandwidth and thermodynamics properties of monolayer graphene in the presence of electrical modulation have been investigated so far[13]. In this work we are interested in studying the effects of electric modulation on magnetoconductivity in bilayer graphene and we compare the results obtained with those of monolayer graphene and the conventional 2DEG.

II. FORMULATION

We consider symmetric bilayer graphene within the single electron approximation described by the following effective Hamiltonian ($\hbar = c = 1$ here)[2, 5]

$$H_0 = -\frac{1}{2m}\begin{pmatrix} 0 & (P_x - iP_y)^2 \\ (P_x + iP_y)^2 & 0 \end{pmatrix}, \tag{1}$$

where $\overrightarrow{p} = -i\overleftrightarrow{\nabla} - e\overleftrightarrow{A}$, with the vector potential expressed in the Landau gauge as $\overleftrightarrow{A} = (0, Bx, 0)$ and the magnetic field is $B = (0, 0, B\check{z})$,which is perpendicular to the bilayer graphene, m is the effective mass of the electrons in bilayer: $m = 0.043m_e$ with m_e the usual electron mass. The energy eigenvalues and eigenfunction in the presence of the magnetic field are

$$\varepsilon(n) = \omega_c\sqrt{n(n-1)}, \quad n \gtrsim 2 \tag{2}$$

where $\omega_c = \frac{eB}{m}$ is the cyclotron frequency. For the low magnetic fields considered in this work, the Hamiltonian of Eq.(1) and the Landau level spectrum in Eq.(2) adequately captures the low energy electronic properties in bilayer in the presence of a magnetic field[5]. The eigenfunction can be written as

$$\Psi^k_{n,K_y}(r) = \frac{e^{iK_y}}{\sqrt{2L_y}}\begin{pmatrix} \Phi_{n-2} \\ \Phi_n \end{pmatrix}, \tag{3}$$

where L_y is the $y-$dimension of the bilayer and the normalized harmonic oscillator eigenfunction are

$$\Phi_n(x) = \frac{1}{\sqrt{2^n n \sqrt{\pi} l}} \exp^{[-\frac{1}{2}(\frac{x-x_0}{l})^2]} H_n(\frac{x+x_0}{l}),$$

with center of the cyclotron orbit $x_0 = l^2 k_y$. We now consider a weak one-dimensional periodic electric modulation in the $x-$direction given by the following Hamiltonian

$$H' = V_0 \cos(Kx), \tag{4}$$

where $K = 2\pi/a$, a is the period of modulation and V_0 is the amplitude of modulation. We apply standard perturbation theory to determine the first order correction to the unmodulated energy eigenvalues in the presence of modulation with the result

$$\varepsilon'(n, x_0) = V_n \cos(Kx_0), \tag{5}$$

where

$$V_n(u) = \frac{V_0}{2} \exp(-u/2)(L_n(u) + L_{n-2}(u)),$$

$u = K^2 l^2/2$,and $L_n(u)$ are Laguerre polynomials.

From equations (2) and (5), the energy eigenvalues for the system in the presence of modulation are

$$\varepsilon(n, x_0) = \omega_c \sqrt{n(n-1)} + V_n \cos(Kx_0). \tag{6}$$

From equation (6) we observe that the formerly sharp Landau levels are now broadened into minibands by the modulation potential. Furthermore, the Landau bandwidth ($\tilde{}| V_n |$) oscillate as a function of n, since $L_n(u)$ is an oscillatory function of its index.

The bandwidth contains an average of Laguerre polynomials with indices n and $n-2$. To compare, in the electrically modulated monolayer graphene the bandwidth depends on a linear combination of Laguerre polynomials with indices n and $n-1$ whereas for standard electrons in 2DEG there is only a single term that contains Laguerre polynomial with index n. We expect that this modulation induced change in the electronic density of states to influence the magnetoconductivity of bilayer graphene and this is calculated in the following section.

III. MAGNETOCONDUCTIVITY WITH PERIODIC ELECTRIC MODULATION

To determine the magnetoconductivity in the presence of weak electric modulation we apply the Kubo formula in the linear response regime. In the presence of the magnetic field, the main contribution to the Weiss oscillations in magnetoconductivity arises from scattering induced migration of the Larmor circle center. This is the diffusive conductivity and we shall determine it following the approach in[10, 11, 12]. In the case of quasielastic scattering of the electrons, the diagonal component σ_{yy} of the conductivity can be calculated by the following expression,

$$\sigma_{yy} = \frac{\beta e^2}{L_x L_y} \sum_{\zeta} f(\varepsilon_\zeta)[1 - f(\varepsilon_\zeta)]\tau(\varepsilon_\zeta)(\upsilon_y^\zeta)^2 \tag{7}$$

L_x, L_y, are the dimensions of the layer, $\beta = \frac{1}{k_B T}$ is the inverse temperature with k_B the Boltzmann constant, $f(\varepsilon)$ is the Fermi Dirac distribution function and $\tau(\varepsilon)$ is the electron relaxation time and ζ denotes the quantum numbers of the electron eigenstate.The diagonal component of the conductivity σ_{yy} is due to modulation induced broadening of Landau bands and hence it carries the effects of modulation in which we are primarily interested in this work. σ_{xx} does not contribute as the component of velocity in the x-direction is zero here. The collisional contribution due to impurities is not taken into account in this work.

The summation in Eq.(7) over the quantum numbers ζ can be written as

$$\frac{1}{A} \sum_{\zeta} = \frac{L_y}{2\pi} \int_0^{\frac{L_x}{l^2}} dk_y \sum_{n=0}^{\infty} = \frac{1}{2\pi l^2} \sum_{n=0}^{\infty} \tag{8}$$

where $A = L_x L_y$ is area of the system. The component of velocity required in Eq.(7) can be calculated from the following expression

$$\upsilon_y^\zeta = \frac{\partial}{\partial k_y} \varepsilon(n, x_0). \tag{9}$$

Substituting the expression for $\varepsilon(n, x_0)$ obtained in Eq.(6) into Eq.(9) yields

$$\upsilon_y^\zeta = \frac{2V_n(u)u}{K} \sin(Kx_0). \tag{10}$$

With the results obtained in Eqs.(8), (9) and (10) we can express the diffusive contribution to the conductivity given by Eq.(7) as

$$\sigma_{yy} = A_0 \phi \tag{11}$$

where

$$A_0 = \frac{2}{\pi} V_0^2 e^2 \tau \beta \tag{12}$$

and the dimensionless conductivity of bilayer graphene ϕ is given as

$$\phi = \frac{ue^{-u}}{4} \sum_{n=0}^{\infty} \frac{g(\varepsilon(n))}{[g(\varepsilon(n)) + 1)]^2} [L_n(u) + L_{n-2}(u)]^2. \tag{13}$$

where $g(\varepsilon) = \exp[\beta(\varepsilon - \varepsilon_F]$ and ε_F is the Fermi energy.

IV. ASYMPTOTIC EXPRESSIONS

To get a better understanding of the results of the previous section we will consider the asymptotic expression of conductivity where analytic results in terms of elementary functions can be obtained[11]. We shall compare the asymptotic results for the dimensionless conductivity obtained in this section with the results obtained for the electrically modulated conventional 2DEG system. We shall also compare these results with recently obtained results for the monolayer graphene that is subjected to only the electric modulation.

The asymptotic expression of dimensionless conductivity can be obtained by using the following asymptotic expression for the Laguerre polynomials

$$\exp^{-u/2} L_n(u) \rightarrow \frac{1}{\sqrt{\pi\sqrt{nu}}} \cos(2\sqrt{nu} - \frac{\pi}{4}). \tag{14}$$

Note that the asymptotic results are valid when many Landau Levels are filled. We now take the continuum limit:

$$n - - > \frac{\varepsilon(n)}{\omega_c}, \sum_{n=0}^{\infty} - - > \int_0^{\infty} \frac{d\varepsilon}{\omega_c} \tag{15}$$

to express the dimensionless conductivity in Eq.(13) as the following integral

$$\phi = \frac{1}{\pi} \int_0^{\infty} d\varepsilon \frac{g(\varepsilon)}{[g(\varepsilon) + 1)]^2} \sqrt{\frac{u}{n}} \cos^2(\sqrt{u/n}) \cos^2(2\sqrt{nu} - \frac{\pi}{4}) \tag{16}$$

where $u = 2\pi^2/b$ and the dimensionless magnetic field b is introduced as $b = \frac{B}{B'}$ with $B' = \frac{1}{ea^2}$.

Now assuming that the temperature is low such that $\beta^{-1} \ll \varepsilon_F$ and replacing $\varepsilon = \varepsilon_F + s\beta^{-1}$, we rewrite the above integral as

$$\phi = \frac{\sqrt{2/\varepsilon_F b\omega_c}}{4\beta} \cos^2\left(\frac{2\pi}{p}\right) \int_{-\infty}^{\infty} \frac{4dse^s}{(e^s + 1)^2} \cos^2\left(\frac{2\pi p}{b} - \frac{\pi}{4} + \frac{4\pi}{p\omega_c} s\right) \tag{17}$$

where $p = k_F a = \sqrt{2\pi n_e} a$ is the dimensionless Fermi momentum of the electron. To obtain an analytic solution we have also replaced ε by ε_F in the above integral except in the sine term in the integrand.

The above expression can be expressed as

$$\phi = \frac{\sqrt{2/\varepsilon_F b\omega_c}}{4\beta} \cos^2\left(\frac{2\pi}{p}\right) \int_{-\infty}^{\infty} \frac{ds}{\cosh^2(s/2)} \cos^2\left(\frac{2\pi p}{b} - \frac{\pi}{4} + \frac{4\pi}{p\omega_c} s\right). \tag{18}$$

The above integration can be performed by using the following identity

$$\int_0^{\infty} dx \frac{\cos ax}{\cosh^2 \beta x} = \frac{a\pi}{2\beta^2 \sinh(a\pi/2\beta)} \tag{19}$$

with the result

$$\phi = \frac{T}{4\pi^2 T_B} \cos^2\left(\frac{2\pi}{p}\right) \left[1 - A\left(\frac{T}{T_B}\right) + 2A\left(\frac{T}{T_B}\right) \cos^2\left[2\pi\left(\frac{p}{b} - \frac{1}{8}\right)\right]\right] \tag{20}$$

where T_B is the characteristic damping temperature of Weiss oscillations in bilayer graphene: $k_B T_B = \frac{bp}{4\pi^2 ma^2}$, $\frac{T}{T_B} = \frac{4\pi^2 ma^2}{bp}$ and $A(x) = \frac{x}{\sinh(x)} -^{(x-->\infty)} - >= 2xe^{-x}$.

V. COMPARISON WITH ELECTRICALLY MODULATED MONOLAYER GRAPHENE

We will now compare the results obtained in this work with results obtained in [11] for the case of electrically modulated monolayer graphene system. We will first compare the energy spectrum in the two cases. The difference in the energy spectrum due to modulation effects was obtained in Eq.(6). If we compare this result with the corresponding expression for the electrically modulated monolayer graphene case, we find the following differences: Firstly, in the monolayer we have an average of two successive Laguerre polynomials with indices n and $n-1$ whereas here we also have the average of two Laguerre polynomials but not successive ones but rather with indices n and $n-2$. Secondly, in the monolayer the energy eigenvalues are multiplied by the square root of the Landau band index $\sqrt{n}$ whereas in the bilayer we have $\sqrt{n(n-1)}$ factor. Thirdly, the cyclotron frequency in the two systems is different since the quasiparticles in monolayer are massless Dirac particles whereas they have a finite mass in the bilayer. These differences cause the velocity expression for the electrons given by Eq.(10) to be different in the two systems.

We now compare the expressions for dimensionless conductivity ϕ given by Eq. (20) with the electrically modulated case (Eq.(22) in [11]). The argument of the cosine terms in the expression for bilayer are $2\pi/p$ whereas in monolayer it is π/p which results in the phase difference of π in the the dimensionless conductivity in the two systems. This we expect as the quasiparticles in graphene (both monolayer and bilayer) are chiral and acquire a Berry's phase in the presence of a magnetic field[1]. The Berry's phase acquired by Dirac electrons in monolayer graphene is π whereas it is 2π for particles in bilayer graphene[2, 5]. Therefore we observe a difference in phase of π in the magnetoconductivity oscillations in the two systems. The dimensionless magnetoconductivity for both electrically modulated mono- and bi-layer graphene as a function of inverse magnetic field is shown in Fig.(1)at temperature $T = 6K$, electron density $n_e = 2.3 \times 10^{11} cm^{-2}$ and period of modulation $a = 350nm$.We also observe that in the region of high magnetic field SdH oscillations are superimposed on the Weiss oscillations. The oscillations are periodic in $1/B$ and the period depends on electron density as $\sqrt{n_e}$.

VI. COMPARISON WITH STANDARD ELECTRON SYSTEM IN 2DEG

We start by comparing the energy spectrum and velocity expression obtained in Eq.(6) and Eq.(10) with similar expressions for the conventional 2DEG where the the quasiparticles are standard electrons [9]. For the energy spectrum, we find that the Landau level spectrum is significantly different from that of standard electrons in conventional 2DEG. The first term $\omega_c\sqrt{n(n-1)}$ in Eq.(6) has to be compared with $\omega_c(n + 1/2)$ with $\omega_c = eB/m_e$ for standard electrons. Not only the dependence on the Landau level index n is different in the two systems but the cyclotron frequency is also not the same due to the difference in mass of the quasiparticles. The modulation effects are carried by the second term where the essential difference is in the structure of the function $V_n(u) = \frac{V_0}{2}\exp(-u/2)(L_n(u) + L_{n-2}(u))$. We find that there is a basic difference: In bilayer we have a sum of two Laguerre polynomials with indices n and $n-2$ whereas only a single Laguerre polynomial appears in the corresponding term for standard electrons in 2DEG. This difference in the $V_n(u)$ function causes the velocity expression for the electrons in bilayer given by Eq.(10) to be different from that of the standard electrons. To highlight the difference in the dimensionless conductivity in the two systems, we compare the asymptotic expression in bilayer Eq.(20) with the corre-

sponding expression for 2DEG (Eq. (25) in [11]). We find that dimensionless conductivity in bilayer has an additional prefactor $\cos^2\left(\frac{2\pi}{p}\right)$ which is not present in the corresponding expression for 2DEG. In addition, conductivity in bilayer contains the characteristic damping temperature T_B which is higher than the corresponding damping temperature in 2DEG T_p due to the smaller effective mass of the quasiparticles in bilayer. This results in the magnetoconductivity oscillations to be more robust with temperature than in 2DEG. To see the effects of this difference on the magnetoconductivity we present the dimensionless magnetoconductivity for both electrically modulated bilayer graphene and the electrically modulated standard 2DEG in Fig.(2),as a function of inverse magnetic field at temperature $T = 6K$, electron density $n_e = 2.3 \times 10^{11} cm^{-2}$ and period of modulation $a = 350nm$. We find that the there is a difference in phase of 2π between the oscillations in magnetoconductivity in the two systems since the quasiparticles in bilayer graphene are chiral. A Berry's phase of 2π is acquired by the quasiparticles in bilayer relative to the standard electrons resulting in the appearence of 2π phase difference in the magnetoconductivity oscillations. We also find a peak in magnetoconductivity in 2DEG that is absent in bilayer which is due to the absence of contribution from the $n = 0$ and $n = 1$ Landau levels as they lie at zero energy.

We also find that the magnetoconductivity oscillations in bilayer graphene are less damped by temperature and are more prounced as compared to those in conventional 2DEG system whereas they are less pronounced and are more damped with temperature compared to those in monolayer graphene. This can be seen in Fig.(3) where dimensionless conductivity as a function of inverse magnetic field is presented for the three systems. The parameters used are: $T = 6K$, electron density $n_e = 2.3 \times 10^{11} cm^{-2}$ and period of modulation $a = 350nm$. This can be understood by considering the temperature scale for damping of Weiss oscillations in bilayer graphene obtained from Eq.(20) which is characterized by T_B given as $k_B T_B = \frac{bp}{4\pi^2 m a^2}$ whereas the characteristic tempereature for 2DEG is given in [10, 11] as $k_B T_p = \frac{bp}{4\pi^2 m_e a^2}$. Comparing T_B and T_p the essential difference is the difference in the effective masses of the quasiparticles in the two systems. Since the quasiparticles in bilayer have a smaller effective mass $m = 0.043 m_e$, the characteristic damping temperature T_B is higher in bilayer than in conventional 2DEG characterized by T_p. Hence Weiss oscillations in magnetoconductivity in bilayer graphene are less damped with temperature compared to 2DEG system.

VII. CONCLUSIONS

We have investigated the diffusive magnetoconductivity component σ_{yy} in bilayer graphene in the presence of a perpendicular magnetic field and a one-dimensional weak electric modulation. In this work, we focus on the Weiss oscillations in magnetoconductivity. We have compared the results obtained with those of electrically modulated monolayer graphene as well as electrically modulated conventional 2DEG system. We find phase differences of π and 2π in the magnetoconductivity oscillations compared to monolayer graphene and conventional 2DEG system which arises due to the chiral nature of quasiparticles in graphene.We also find that the oscillations due to modulation in the magnetoconductivity are enhanced and less damped with temperature compared to conventional 2DEG system whereas they are less robust with temperature compared to monolayer graphene.

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∗Present address: The Blackett Laboratory, Imperial College London, SW7 2AZ London, United Kingdom.

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